# Quadratic Equation

## Question

**1.** Solve for x :

$$\frac{x^2-3}{k^2-3}=\frac{2x+1}{2k+1}$$

**2.** Solve for x :

$$\left(x^2-5x+5\right)^{(x^2-9x+20)}=1$$

**3.** Solve for x:

$$\left(\sqrt{2}+1\right)x^2-\left(\sqrt{2}+3\right)x+\sqrt{2}=0$$

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**4.** The quadratic equation:

$$x^2 + 3x + a = 0$$

has two integral solutions.

If $a \ge 0$, solve the equation.

**5.** Solve the quadratic equation:

$$x^2-2x-15+\frac{36}{x^2-2x}=0$$

6. Solve the equation:

$$x^3-2\sqrt{2}\,x^2+2x-\sqrt{2}+1=0$$

(Hint : Put $y=\sqrt{2}$ )

<u>**Solution**</u>

**1.** Obviously, by substitution

$x = k$ is a root.

The given equation is a quadratic equation.

$(x^2 - 3)(2k + 1) = (2x + 1)(k^2 - 3)$

$(2k + 1)x^2 - 2(k^2 - 3)x - (k^2 + 6k) = 0$ (*)

Product of roots = $-\dfrac{k^2 + 6k}{2k+1} = k\left(-\dfrac{k+6}{2k+1}\right)$

$\therefore$ **The roots are:** $\mathbf{x = k,\ -\dfrac{k+6}{2k+1}}$.

Note : If you factorize (*), you can get:

$[(2k + 1)x + (k + 6)][x - k] = 0$

**2.** Since **(a)** $1^k = 1$ and

**(b)** $(-1)^{2n} = 1$, where n is an integer.

**(c)** $x^0 = 1,\ x \neq 0$

**(a)** If $x^2 - 5x + 5 = 1$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$ $\therefore$ $\mathbf{x = 1, 4}$

**(b)** If $x^2 - 5x + 5 = -1$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$ $\therefore$ $\mathbf{x = 2, 3}$

(i) When $x = 2$, $x^2 - 9x + 20 = 6$

(ii) When $x = 3$, $x^2 - 9x + 20 = 2$

In both cases, $x^2 - 9x + 20$ are even.

**(c)** $x^2 - 9x + 20 = 0$

$(x - 4)(x - 5) = 0$ $\therefore$ $\mathbf{x = 4, 5}$

(i) When $x = 4$, $x^2 - 5x + 5 = 1 \neq 0$

(ii) When $x = 5$, $x^2 - 5x + 5 = 5 \neq 0$

$\therefore$ **The solutions are:** $\mathbf{x = 1, 2, 3, 4, 5.}$

**3.** Multiply the given equation by $(\sqrt{2}-1)$:

$$x^2+\left(1-2\sqrt{2}\right)x+\sqrt{2}\left(\sqrt{2}-1\right)=0$$

On factorization, we get:

$$\left(x-\sqrt{2}\right)\left[x-\left(\sqrt{2}-1\right)\right]=0$$

$\therefore\quad x=\sqrt{2}$ or $\sqrt{2}-1$

**4.** The given equation has <u>real</u> solution.

$\therefore\ \Delta = 9 - 4a \geq 0$

$\therefore\ a \leq 9/4 \qquad (1)$

Let $\alpha, \beta$ be the roots of the equation.

By Vieta Theorem,

$\alpha + \beta = -3 \qquad (2)$

$\alpha\beta = a \qquad (3)$

From (3), a is an integer.

From (1) and $a \geq 0$ (given),

$a = 0, 1, 2.$

(i) When $a = 0$, from (2) and (3)

$\alpha + \beta = -3,\ \alpha\beta = 0$

The roots are $-3, 0$.

(ii) When $a = 1$, from (2) and (3)

$\alpha + \beta = -3,\ \alpha\beta = 1$

There is no integral roots.

(iii) When $a = 2$, from (2) and (3)

$\alpha + \beta = -3,\ \alpha\beta = 2$

The roots are $-1, -2$.

$\therefore$ The roots are $-3, -2, -1, 0$.

**5.** $$x^2 - 2x - 15 + \frac{36}{x^2 - 2x} = 0 \qquad (1)$$

Put $y = x^2 - 2x$,

(1) becomes:

$$y - 15 + \frac{36}{y} = 0$$

$y^2 - 15y + 36 = 0$

$(y - 3)(y - 12) = 0$

$\therefore y = 3, 12$

$x^2 - 2x = 3$ , $\quad x^2 - 2x = 12$

$x^2 - 2x - 3 = 0$, $\quad x^2 - 2x - 12 = 0$

$(x - 3)(x + 1) = 0$, $x^2 - 2x - 12 = 0$

**x = 3, -1 or** $\mathbf{x = 1 \pm \sqrt{13}}$

**6.** $$x^3 - 2\sqrt{2}\,x^2 + 2x - \sqrt{2} + 1 = 0$$

Put $y = \sqrt{2}$

$x^3 - 2yx^2 + y^2x - y + 1 = 0$

$xy^2 - (2x^2 + 1)y + (x^3 + 1) = 0$

which is a quadratic equation in y.

$[y - (1 + x)][xy - (1 - x + x^2)] = 0$

(You may also use quadratic equation formula)

Since $x \neq 0$,

$$\therefore y = 1 + x \qquad \text{or} \qquad y = \frac{1 - x + x^2}{x}$$

$x = y - 1 \quad$ or $\quad xy = 1 - x + x^2$

$x = y - 1 \quad$ or $\quad x^2 - (y + 1)x + 1 = 0$

$$x = y - 1 \quad \text{or} \quad x = \frac{y + 1 \pm \sqrt{(y+1)^2 - 4}}{2}$$

$$\therefore x = \sqrt{2} - 1 \quad \text{or} \quad x = \frac{\sqrt{2} + 1 \pm \sqrt{2\sqrt{2} - 1}}{2}$$